Scale resolved analysis of brain functional connectivity networks with spectral entropies

Carlo Nicolini

carlo.nicolini@iit.it

Center for Neuroscience and Cognitive Systems Istituto Italiano di Tecnologia



Resting state brain networks



Fox et al. 2005



Greicius et al. 2007

- Spontaneuos fluctuations of BOLD fMRI signal
- Subject at rest, 5 minutes.
- Effects of motion.
- Effects of thresholding.
- Effects of diseases.

Graph theory to access information in rs-fMRI



Continuous association measure (correlation, mutual information, coherence etc...)

How to compare brain networks?

- Weights are positive and negative.
- Matrices are dense (computationally hard).
- Spurious correlations (e.g. motion).



Before thresholding After thresholding After thresholding Appropriate null models are needed

- Maximally non-committal to missing information.
- How far are networks from random?
- Detection of significant structural patterns in real networks.
- Maximum Entropy Random Graphs (classical)
- Spectral entropies (quantum)

Probability distributions on graphs



Maximize entropy $S(\mathcal{G}) = -\sum_{G \in \mathcal{G}} P(G) \ln P(G)$ with constraints.

$$P(G) = \frac{e^{-H}}{Z}$$

Continuous Enhanced Configuration model

- Degree sequence $k_i = \sum_j \Theta(w_{ij} t)$
- Strength sequence $s_i = \sum_j w_{ij} \Theta(w_{ij} t)$

Hamiltonian: 2n parameters

$$H(G|\boldsymbol{\alpha},\boldsymbol{\beta}) = \sum_{i < j} \alpha_{ij} \Theta(w_{ij} - t) + \beta_{ij} w_{ij} \Theta(w_{ij} - t)$$



Spectral entropies and diffusion on networks



Von Neumann entropy and density

$$S(\boldsymbol{\rho}) = -\operatorname{Tr}\left[\boldsymbol{\rho}\ln\boldsymbol{\rho}\right] \qquad \boldsymbol{\rho} = \frac{e^{-\beta \mathbf{L}}}{\operatorname{Tr}\left[e^{-\beta \mathbf{L}}\right]}$$

Shaped by Laplacian eigenvalues (spectral gap) Von Neumann relative entropy: **scale resolved network comparison**

$$S(\boldsymbol{\rho} \| \boldsymbol{\sigma}) = \operatorname{Tr} \left[\boldsymbol{\rho} \left(\ln \boldsymbol{\rho} - \ln \boldsymbol{\sigma} \right) \right]$$

Network thresholding

- Sparsification to reduce effects of experimental noise.
- Computationally tractable.
- rs-fMRI networks, N=638, 24 healthy subjects.



Effects of network thresholding



Disruption of modular architecture in schizophrenia



¹Bordier et al. Neuroimage: Clinical (2018)

Effects of disease: schizophrenia



Motion and its correction

- Long range spurious correlation
- Pipelines for correction
- Aggressive vs. conservative

Effects of motion





Conclusions

- rs-fMRI null models
- X
- y